Local low rank denoising for high resolution fMRI

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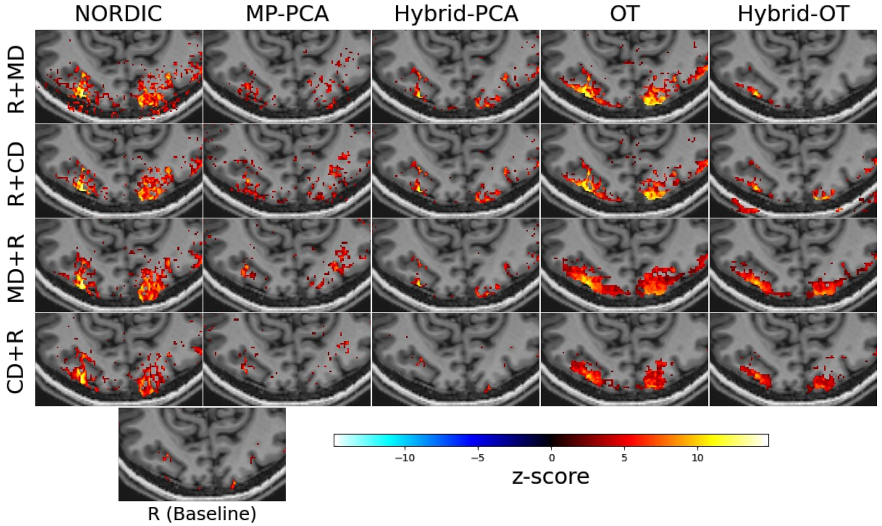
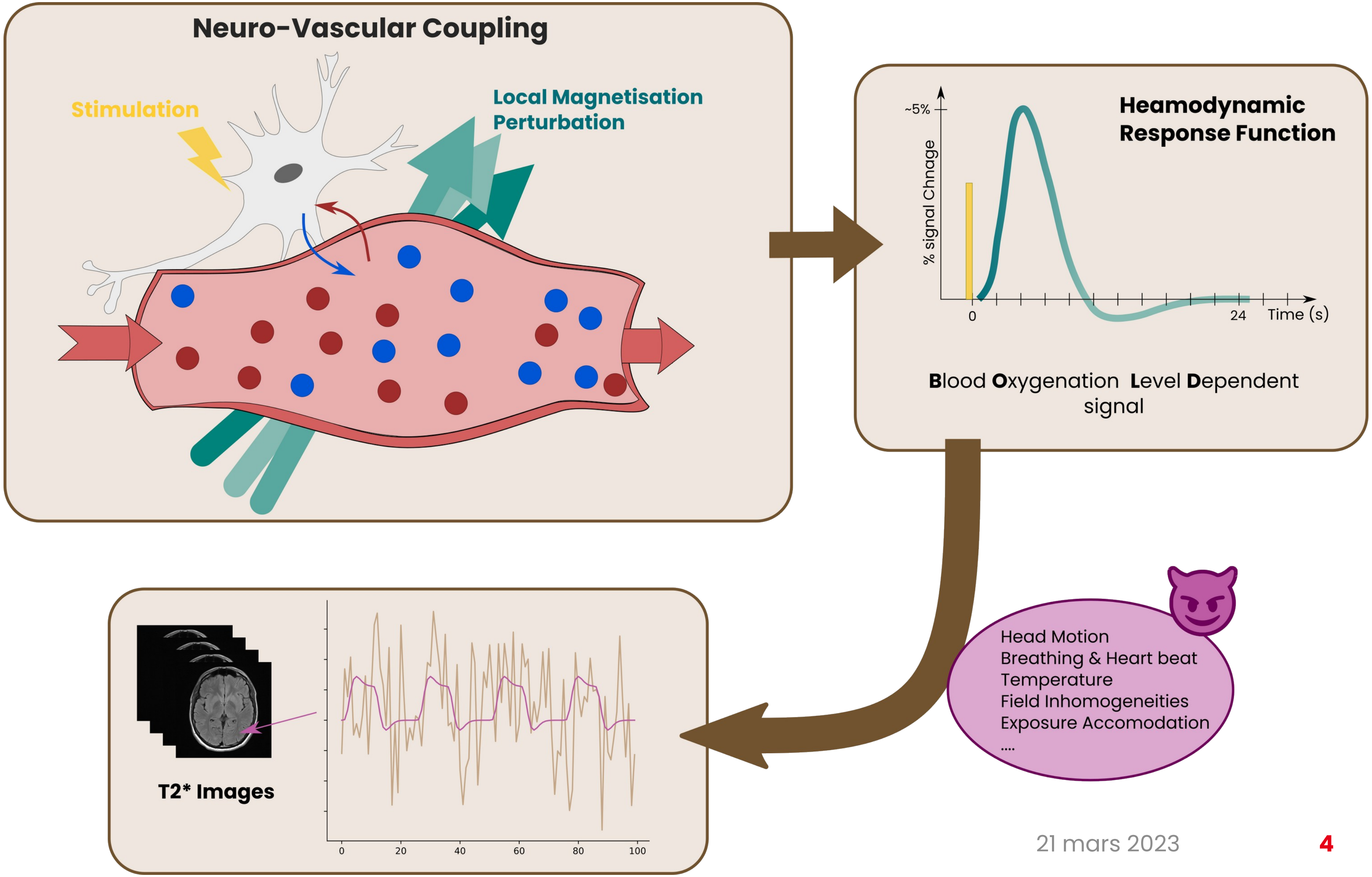


Outline

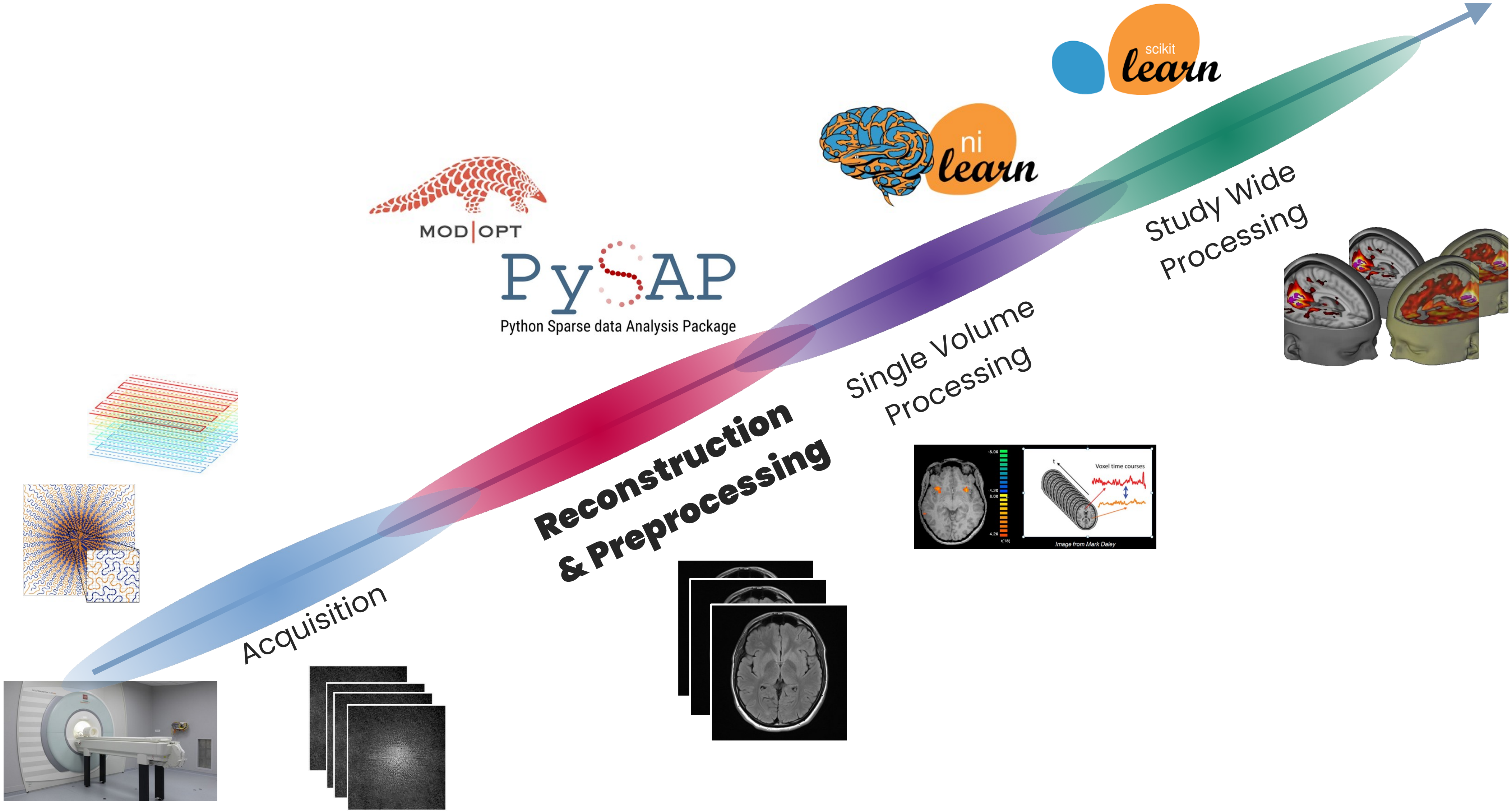
1. fMRI Physics and Processing

The needle in the haystack



Ogawa et al. (1990)
Glover et al. (1997)
… and a lot more

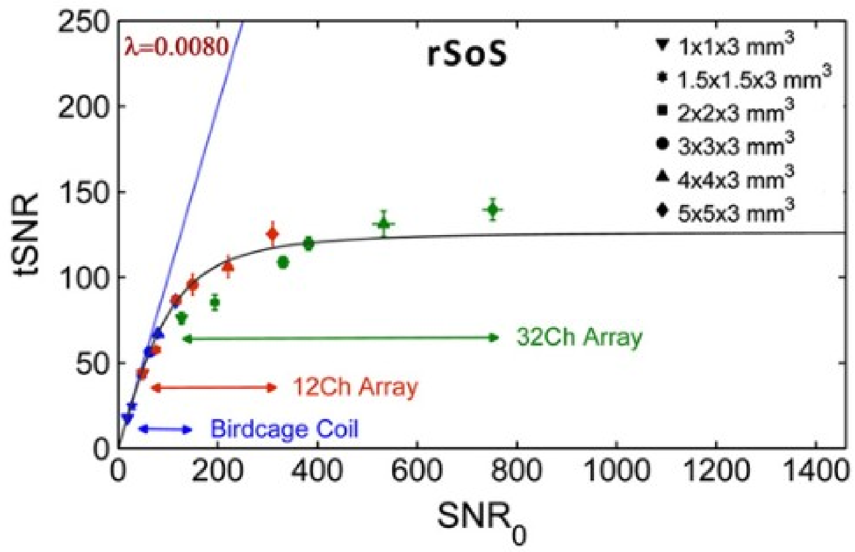
The long way from data to results



The signal of interest and SNR

$$\mathrm{SNR} \propto \frac{B_0^{1.94}(\Delta x)^3}{g\sqrt{R}}$$

$$\mathrm{tSNR} \hat{=} \frac{\langle v \rangle_t}{\sqrt{\langle (v - \langle v \rangle_t)^2 \rangle_t}} \simeq \frac{\mathrm{SNR}}{\sqrt{1 + \lambda^2 \mathrm{SNR}^2}}$$



[Le Ster et al. 2022]

[Triantafyllou et al. 2011]

- Best Image Quality (SNR) does not imply best statistical analysis
- Two Main Source of Noise
  - Thermal noise (Complex Gaussian)
  - Physiological Noise (Breathing, heart beat, motion …)
- Physiological Noise can be dealt with using GLM regressors and external acquisition

At high resolution (= low Spatial SNR), Noise in fMRI data is mostly thermal

2 ▪ Local low-rank methods: A review

A recent buzz on a old method

2688

Hybrid PCA denoising - improving PCA denoising in the presence of spatial correlations

Rafael Neto Henriques[1], Sune Nørhøj Jespersen[2,3], and Noam Shemesh[1]
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Synopsis

PCA denoising based on the Marchenko-P
dimensional MRI. Here we developed a Hy
eigenvalue classification, to overcome sho
classify the noise PCA components in data
PCA denoising can thus be a useful proce

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Lowering the thermal noise barrier in functional brain mapping with magnetic resonance imaging

Luca Vizioli, Steen Moeller, Logan Dowdle, Mehmet Akçakaya, Federico De Martino, Essa Yacoub & Kamil Uğurbil

Nature Communications 12, Article number: 5181 (2021) | Cite this article

Noise Reduction in BOLD-Based fMRI Using Component Analysis

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1100

Optimal Singular-Value Shrinkage for fMRI Denoising

Mo Shahdloo[1] and Mark Chiew[1]
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Synopsis

Singular-value truncation techniques have shown promise for reducing thermal noise in fMRI, where singular-values below a certain threshold are assumed to be noise and are discarded. However, this approach could lead to suboptimal signal recovery, since the remaining singular-values could still have variance contributed by noise. Here instead we propose to use a theoretically MSE-optimal function to shrink the remaining singular-values. The proposed method is evaluated using simulations and high-resolution in-vivo human brain data, and is shown to improve signal-to-noise ratio and functional statistics while leaving the spatial precision intact.

3326

NORDIC PCA increases tSNR in both human and mouse resting-state fMRI for potential improvements in cerebrovascular reactivity mapping

Emily L. Tse[1], Russell W. Chan[1,2], Sarah Y. Wu[1], Yixi Xue[1], Peiying Liu[3], Steen Moeller[4], and Kevin C. Chan[1,5]
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Denoise Functional Magnetic Resonance Imaging With Random Matrix Theory Based Principal Component Analysis

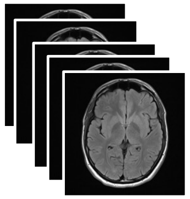
Wei Zhu, Xiaodong Ma, Xiao-Hong Zhu, Kamil Ugurbil, Wei Chen, and Xiaoping Wu

Global to local low rank

- Let the sequence of fMRI Images:

$$\mathbf{Y} = \mathbf{X} + \mathbf{N} \in \mathbb{C}^{N_x N_y N_z \times N_t} \quad N_{ij} \sim \mathcal{N}(0, \mathbf{\Sigma}^2)$$

- Denoising using a low-rank *a priori*
  - fMRI Image share a lot of common information

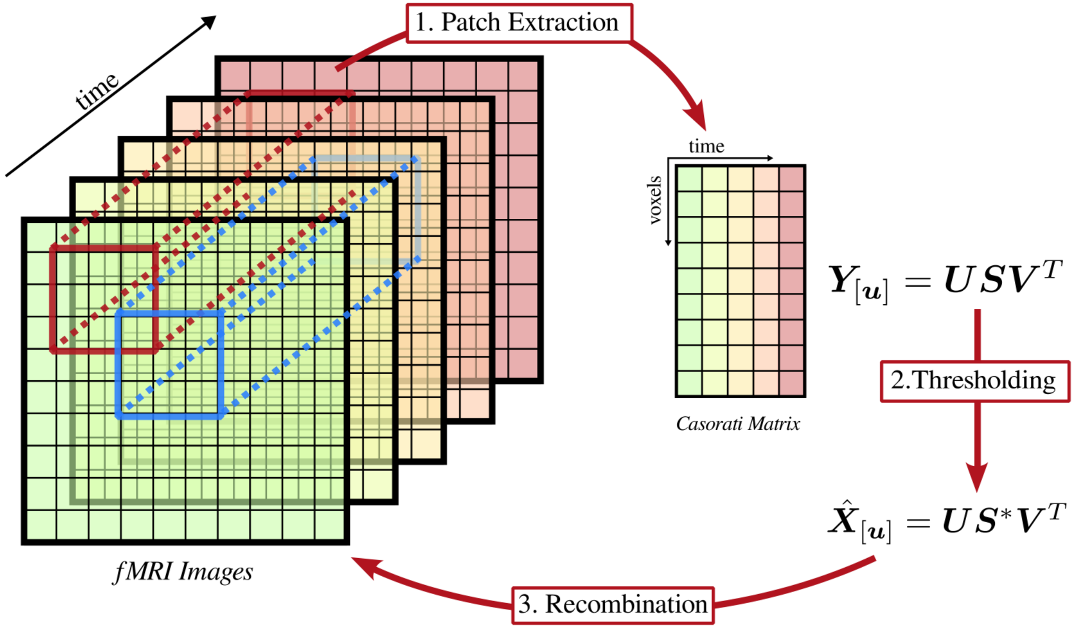


$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \frac{1}{2} \|\mathbf{Y} - \mathbf{X}\|_F^2 + \lambda \|\mathbf{X}\|_*$$

    - $N_x N_y N_z \sim 10^5$ N_t : **DoF limited for the rank constraint**
    - Noise Level is **spatially heterogeneous**

→ Use a *Local* formalism

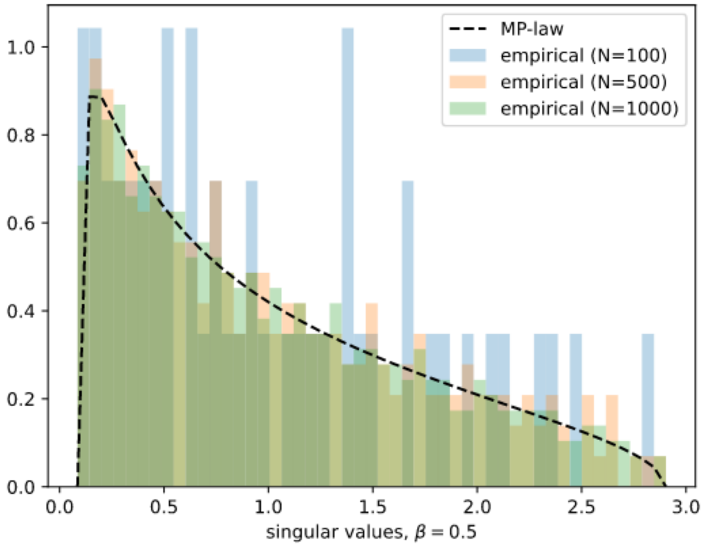
$$\mathbf{Y}_{[\mathbf{u}]} = \mathbf{X}_{[\mathbf{u}]} + \mathcal{N}(0, \sigma_{[\mathbf{u}]})$$



Marchenko-Pastur Law

- Hypothesis

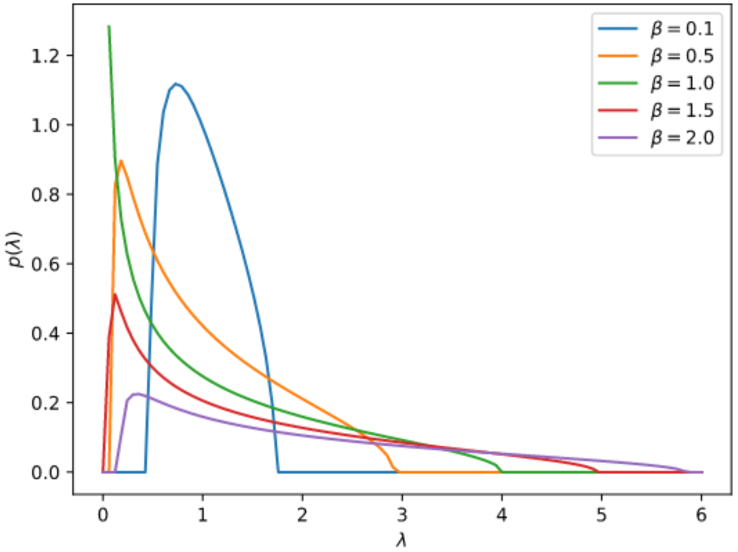
$$\mathbf{X}_N \in \mathbb{C}^{\beta N \times N} \hookrightarrow \mathcal{N}(0, \sigma^2)$$



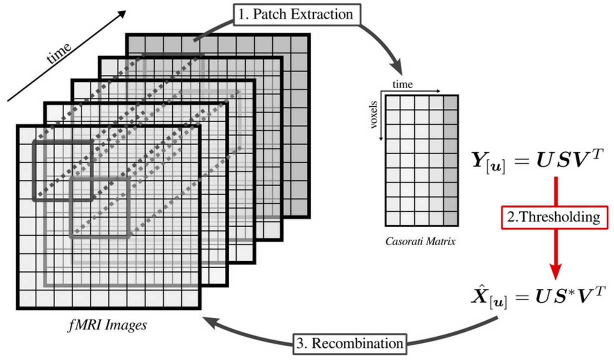
- Results $N \to \infty$
  - The eigenvalues of $\mathbf{Y}_N = \frac{1}{N}\mathbf{X}\mathbf{X}^T$ follows the distribution:

$$p(\lambda|\sigma,\beta) = \frac{\sqrt{(\beta_+ - \lambda)(\lambda - \beta_-)}}{2\pi\beta\lambda\sigma^2}\mathbf{1}_{\lambda\in[\beta_-,\beta_+]}$$

  - Support $\beta_\pm = \sigma^2(1 \pm \sqrt{\beta})^2$
  - Expectation $\mathbb{E}[\lambda] = \sigma^2$



Singular Value Thresholding



- Solution of

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \frac{1}{2}\|\mathbf{Y} - \mathbf{X}\|_F^2 + \lambda\|\mathbf{X}\|_*$$

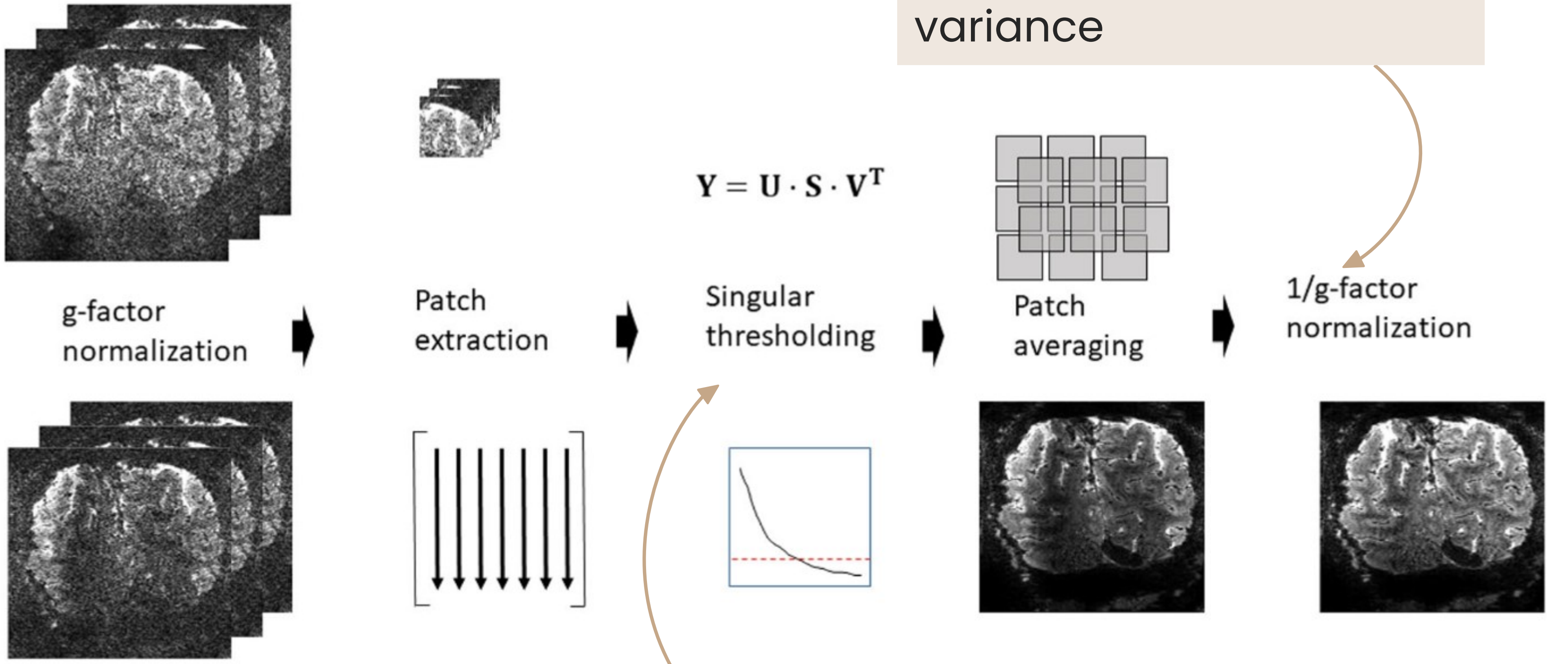
- Given by the **singular value hard/soft thresholding**

$$\mathbf{Y} = \mathbf{USV}^T = \sum_{i=1}^{N} s_i \mathbf{u}_i \mathbf{v}_i^T \longrightarrow \hat{\mathbf{X}} = \sum_{i=1}^{N} \eta(s_i)\mathbf{u}_i \mathbf{v}_i^T$$

- Main Challenges
  - Choice of the threshold (noise level dependent)
    - Estimation of Noise Variance
  - Efficient Computation (lots of patches)
- Highlight two methods
  - **NORDIC**
  - **Optimal Threshold**

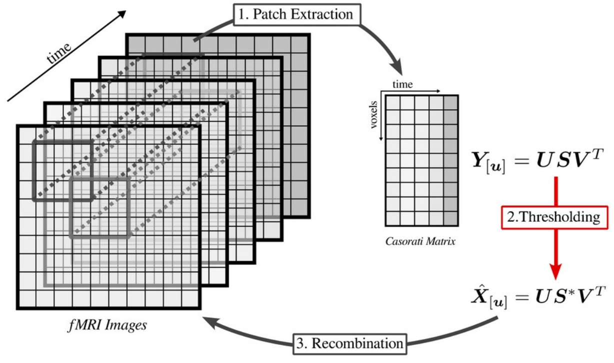
NORDIC : An empirical approach

g-factor: Estimated noise variance



Threshold: largest singular value of a *simulated* noise-only patch

Fig. 7 of [Viozili et al. 2021]
NORDIC Workflow
(Annotations added).



Optimal Threshold : A Mathematical approach

Mathematical Formulation

$$\mathbf{Y} = \mathbf{X} + \mathbf{Z}/\sqrt{N}$$

Data
$\mathbf{Y} \in \mathbb{C}^{M\times N}$
$\beta = M/N$

Low Rank Matrix *Unknown*

iid Gaussian Matrix

Find the best low rank matrix:

$$\arg\min_{\hat{\mathbf{X}}} \|\mathbf{X} - \hat{\mathbf{X}}\|_2^2$$

Classical Solution:

Truncated SVD

$$\mathbf{Y} = \mathbf{USV}^T = \sum_{i=1}^{N} s_i \mathbf{u}_i {\mathbf{v}_i}^T$$

Solution:
LR approx.

$$\hat{\mathbf{X}} = \sum_{i=1}^{N} \eta(s_i)\mathbf{u}_i \mathbf{v}_i^T$$

Thresholding Functions

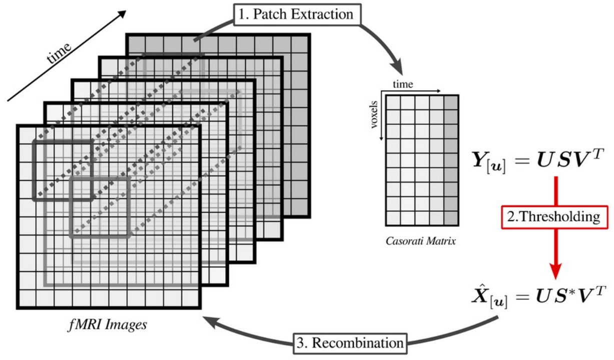
Hard $\eta(s) = s 1_{s>\theta}$

Soft $\eta(s) = \max(0, s-\theta)$

Optimal θ
[Gavish et al. 2014]

Optimal
[Gavish et al. 2017]

$$\eta^*(s) = \begin{cases} \frac{1}{s}\sqrt{(s^2-\beta-1)^2 - 4\beta} & s \geq 1+\sqrt{\beta} \\ 0 & s \leq 1+\sqrt{\beta} \end{cases}$$



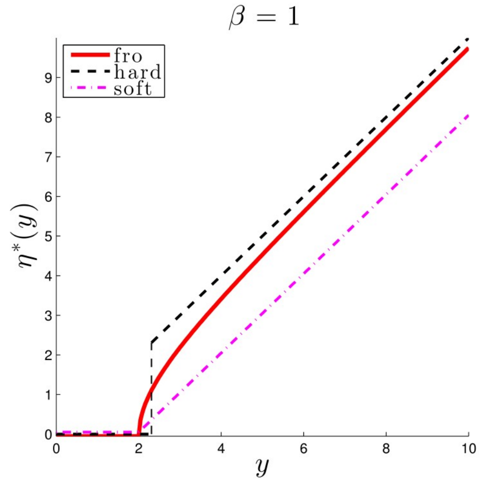
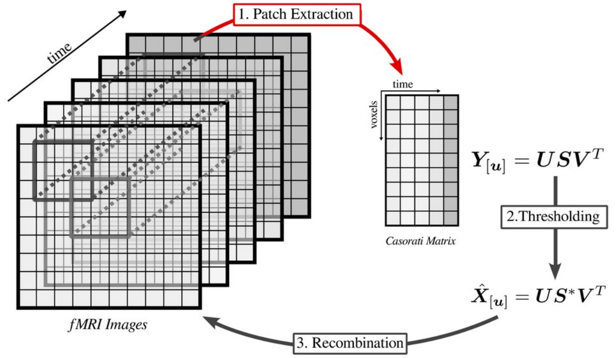


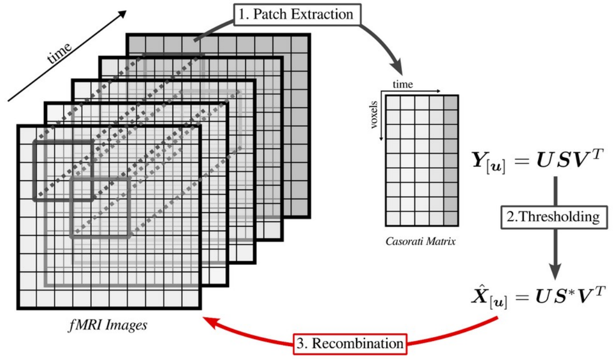
Fig 2 of [Gavish and Donoho, 2014]
Hard, Soft and Optimal Thresholding function

Speed Trick : Use a mask



- Configuration of patches defined by
  - Shape (eg. 11x11x11)
  - Overlap (eg 5 px in each dimensions)
- The problem complexity is O(N_patches x SVD)
  - Reduces the patches by de-noising only a ROI
  - Up to 2/3 of patches discarded in 3D !
- Using a rough mask of the brain
  - Otzu’s binary thresholding + Convex hull
  - Only process patch if at least 50% contains mask

Recombination Strategies



1) Average of Patches

2) Weighted Average of patches *[Majon et al. 2013]*

$$\hat{\mathbf{X}}_i = \frac{\sum_{j=1}^{P} w_j \mathbf{X}_{[\mathbf{u}_j]}}{\sum_{j=1}^{P} w_j} \qquad w_j = \frac{1}{1 + \|S^*_{[\mathbf{u}_j]}\|_0}$$

- Lower rank patches are promoted

3) Select Center value of patches

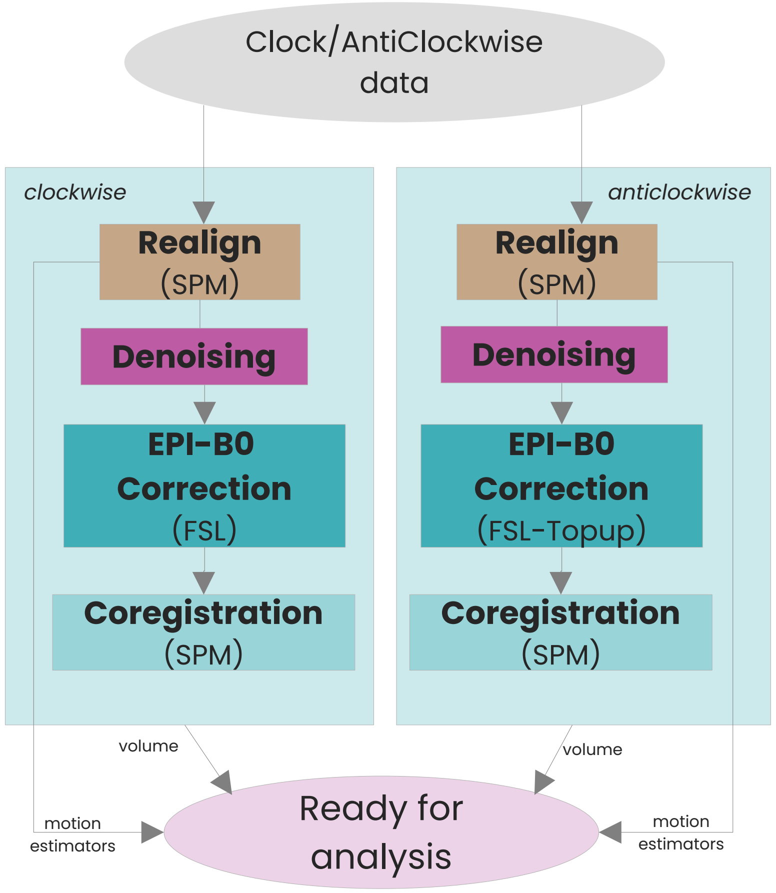
- Requires a maximum overlap

3 Local low-rank methods: Results

Materials and Methods

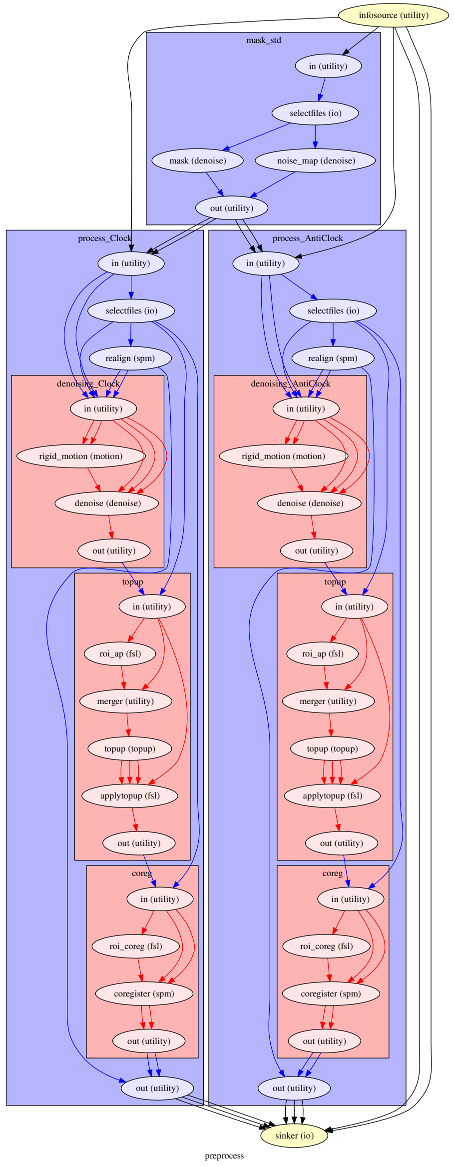
- Retinotopy Task
  - 1mmiso TR=2.4s
  - 3D-EPI Sequence
  - *6 Subjects*
  - *External noise map available*
- Same LLR Parameters
  - Patch size: 11 x 11 x 11
  - Patch overlap: 50%
  - mask and weighted average
- Statistical test:
  - F-test on global effect
- **4** Scenarios (rows)
  - *Baseline: Realign Only*
  - **(R + MD)** Realignment Magnitude Denoising
  - **(R +CD)** Realignment, Complex image Denosing
  - **(MD+R)** Magnitude Denoising, Realignment
  - **(CD+R)** Complex image Denoising, Realignment
- **5 Methods** (columns)
  - ***NORDIC***
  - **MP-PCA** (DMRI Method)
  - **Hybrid-PCA** (Use an external noise map)
  - **Optimal-Threshold**
  - **Hybrid-OT** (Use an external noise map)

Methods : Preprocessing framework



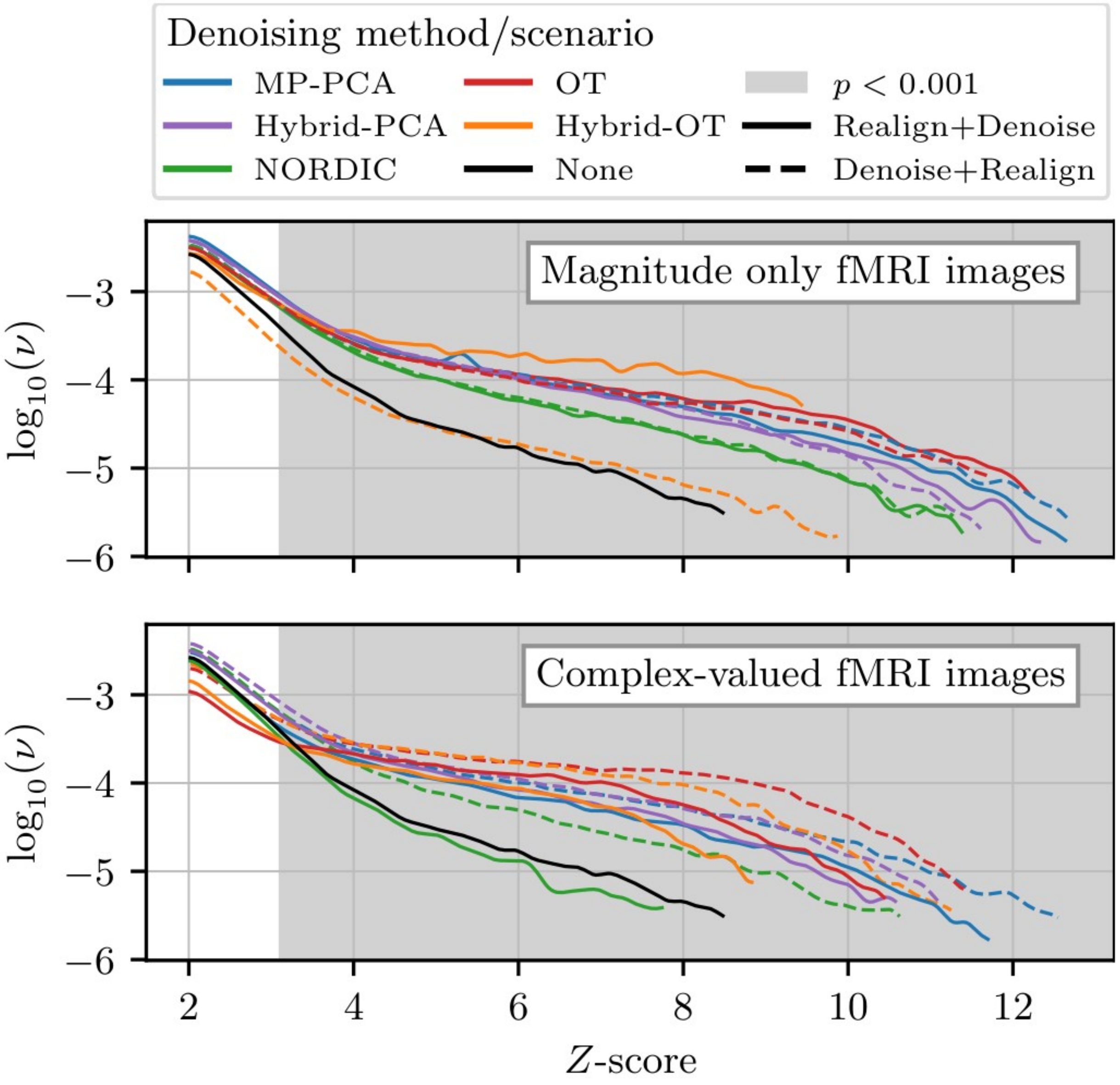
Nipype

- Automatic call to SOTA librairies (FSL, SPM, , …)
- Ordering and Caching
- Multi processing

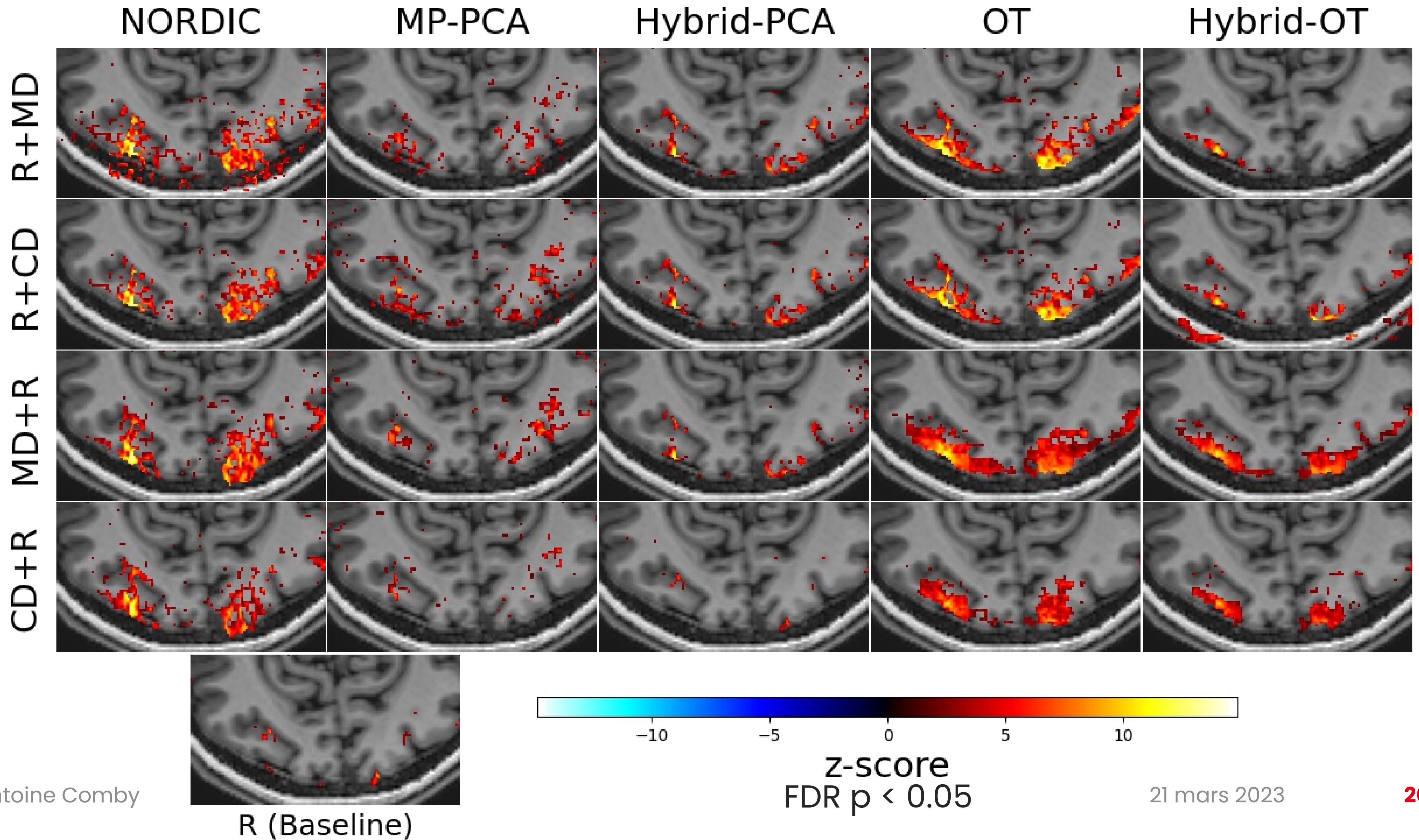


A Boost on Z-Score

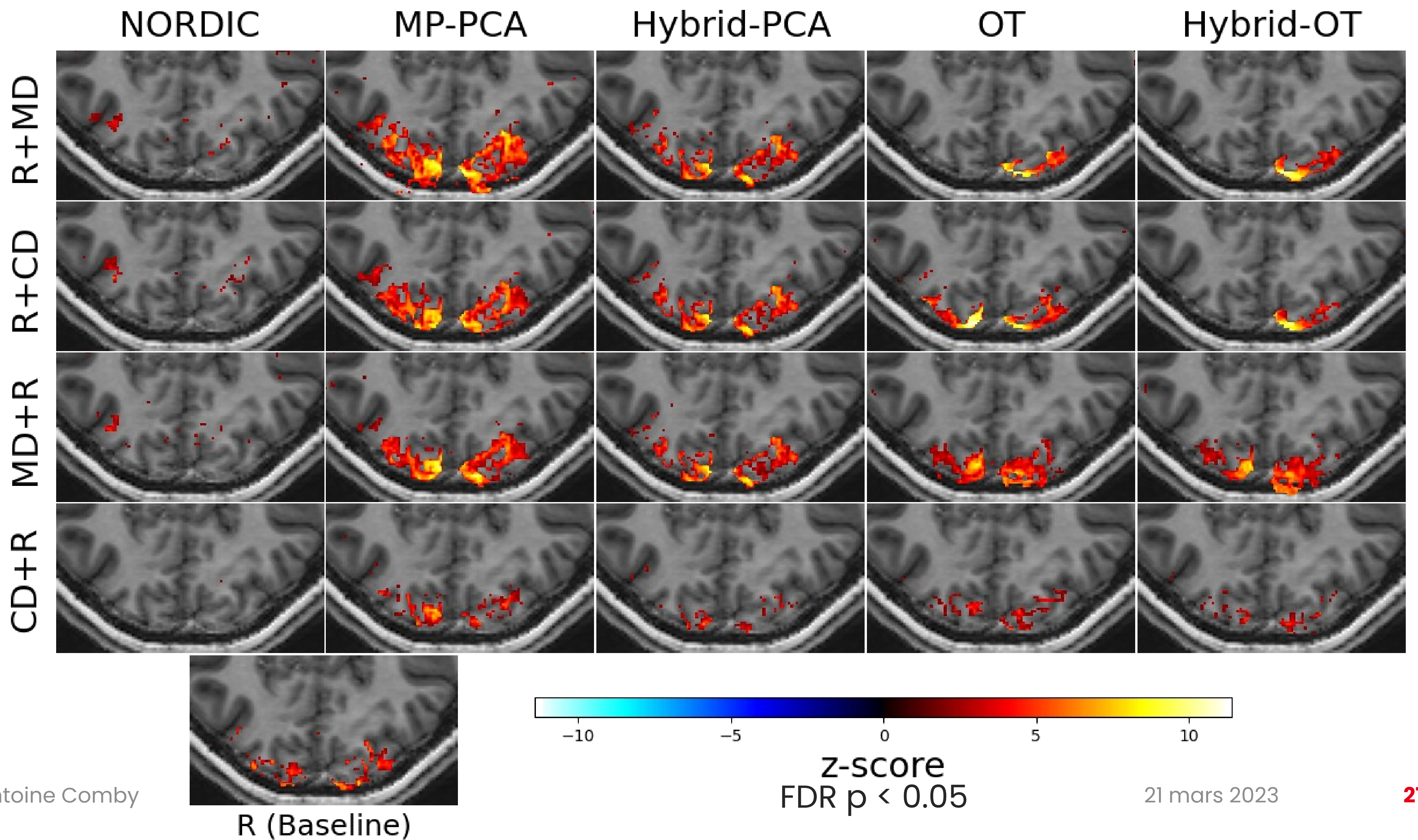
- Compare to baseline (no de-noising), all denoising methods shift the z-score distribution $\nu(Z)$
- Complex Image de-noising perform best when denoising is done after motion correction
  - Motion Correction is a trade-off between reinterpolation of the data (changing noise statistics) and lower rank signals



Activation Map (subject 1)



Activation Map (subject 3)



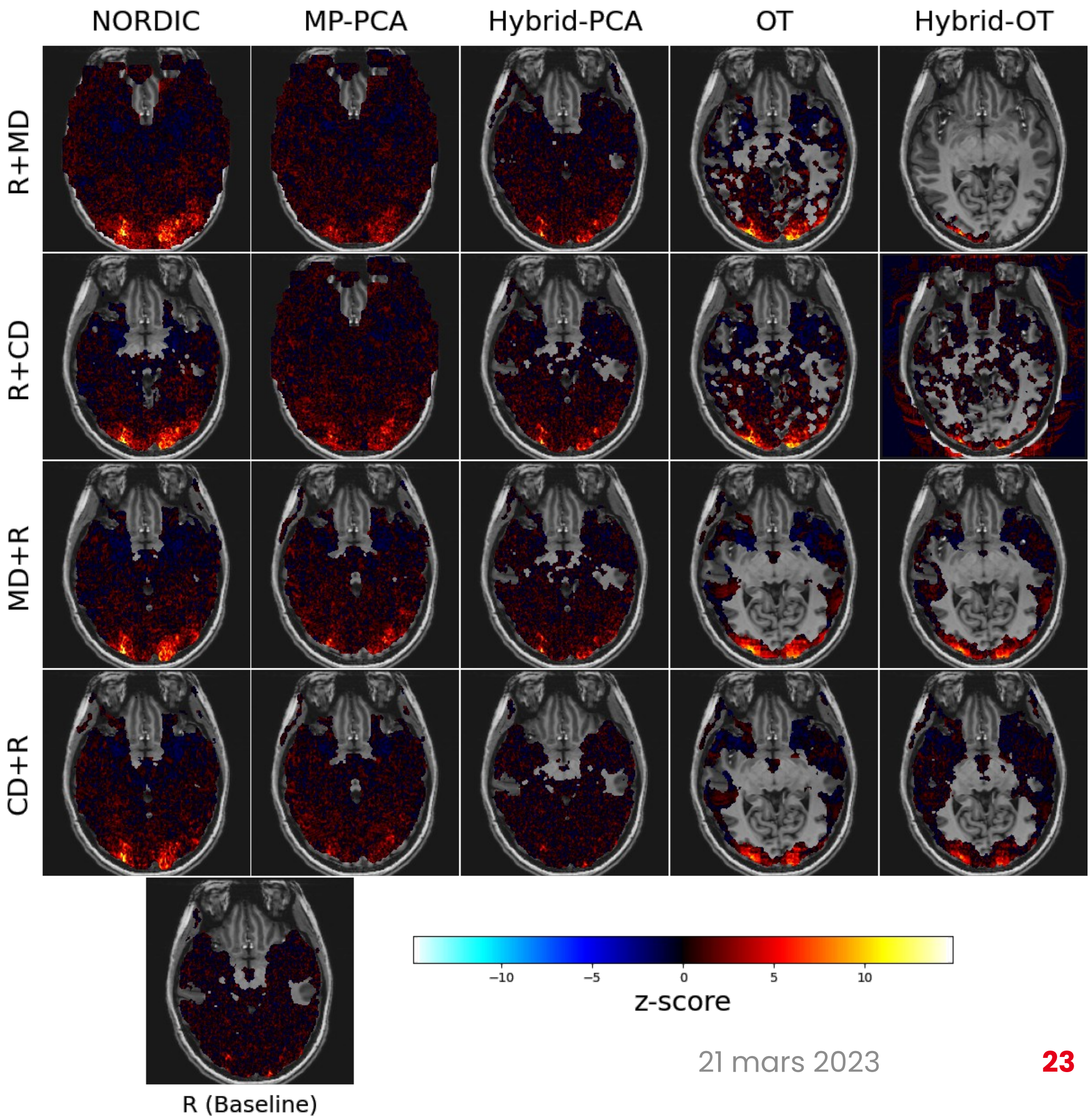
« Group-level » Study

Denoiser	NORDIC	MP-PCA	Hybrid-PCA	OT	Hybrid-OT
R+MD	$\times3.52$ $\times3.64$	$\times6.02$ $\times4.92$	$\times\mathbf{6.09}$ $\times\mathbf{4.93}$	$\times3.73$ $\times4.33$	$\times0.91$ $\times1.00$
R+CD	$\times0.57$ $\times0.55$	$\times2.98$ $\times2.70$	$\times3.27$ $\times2.91$	$\boxed{\times\mathbf{8.03}}$ $\boxed{\times\mathbf{5.04}}$	$\times5.29$ $\times4.01$
MD+R	$\times3.36$ $\times3.48$	$\times6.32$ $\times4.77$	$\times\mathbf{7.55}$ $\times\mathbf{6.10}$	$\times3.10$ $\times3.19$	$\times1.22$ $\times4.90$
CD+R	$\times2.57$ $\times2.59$	$\times5.53$ $\times4.39$	$\times4.97$ $\times4.27$	$\times\mathbf{7.91}$ $\times\mathbf{6.26}$	$\times5.45$ $\times5.00$

Average Detection Gain (FDR $p<0.05$)
in Whole brain (top) and in ROI (bottom)

Local low-rank methods : An interesting Property

- Remove variability in White Matter
  - Best method for this is OT
- Preserve Contrast in Gray Matter
- Overall boost of z-scores



Local low-rank methods: Please use it !

- Python package available

```
pip install patch-denoise
```

- Command line Interface
  - `patch-denoise input.nii output.nii --mask=auto --config=optimal-fro_11_5_w`
- Also available
  - Nipype Interface
  - Functional API

```
denoised, noise_map, weights = optimal-thresholding(noisy, 11, 5, ...)
```

- Documentation and code available here:
  - https://github.com/paquiteau/patch-denoising
  - Stars, issues, PR always welcome

Conclusion & Future work

- High Resolution fMRI benefits from de-noising: **8x increase in voxel activations**
- Complex-valued data is not mandatory to get better results.
- Local low-rank methods are now easily accessible for fMRI and other contrasts
  - Early results for CEST Imaging at Neurospin
- Future Work
  - Add other methods to the package (NLM, SURE thresholding)
  - Faster Computation with GPU processing
  - Application to other Paradigms and Constrast
    - First results in CEST imaging
    - Resting State data

Thank you !

Questions ?

- https://hal.science/hal-03895194
- https://github.com/paquiteau/simfmri
- https://github.com/paquiteau/pysap-fmri
- https://github.com/paquiteau/patch-denoising
- https://github.com/paquiteau/mri-nufft

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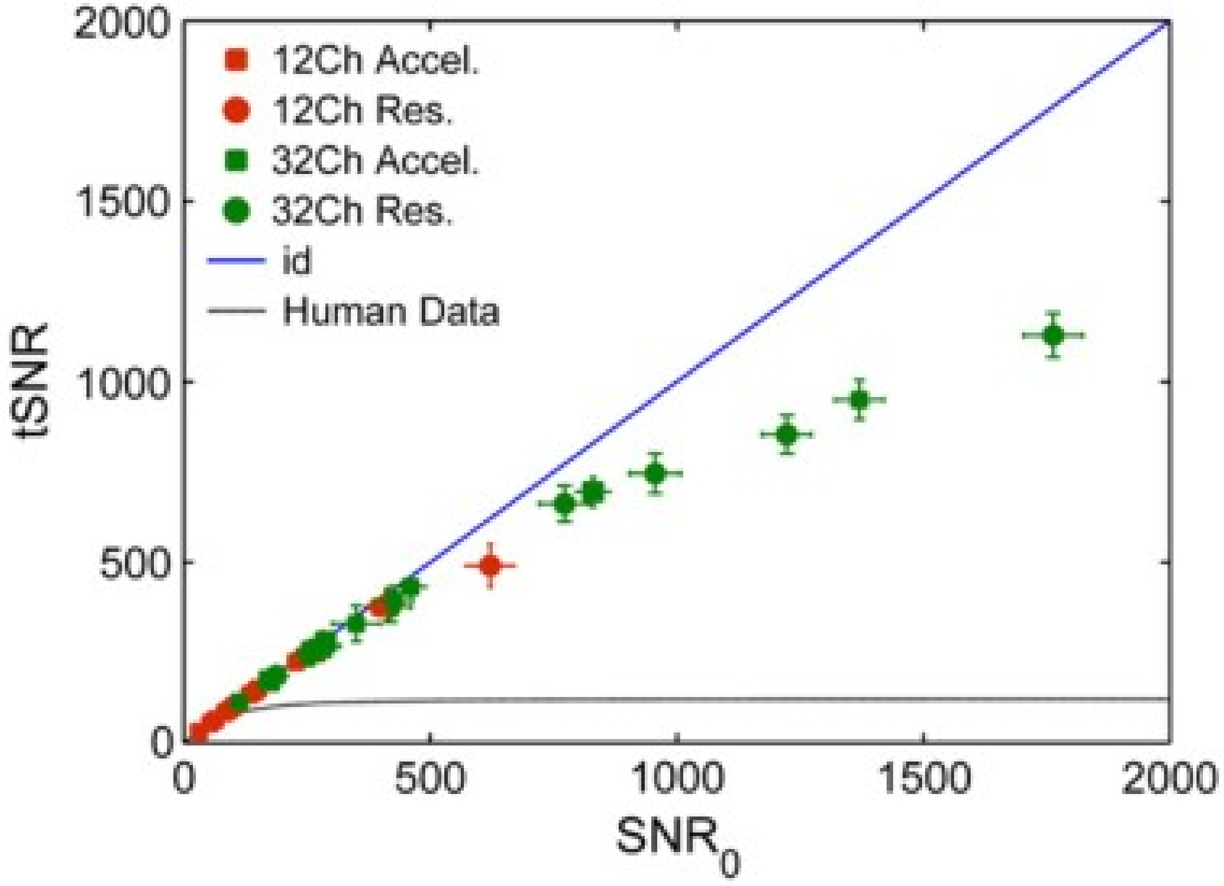
More on tSNR vs SNR



[Triantafyllou et al. 2011]